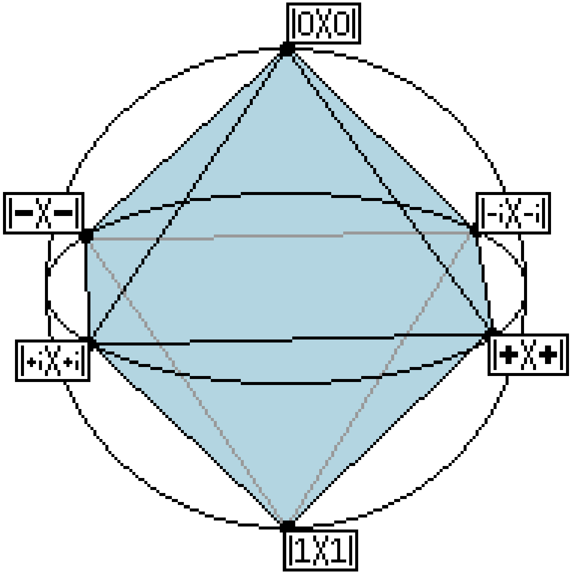
Simulating Quantum Circuits by Shuffling Paulis

Patrick Rall

Aug 21, 2018

The University of Texas at Austin
Quantum Information Center

Shuffling Stabilizer States



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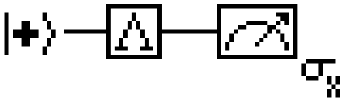

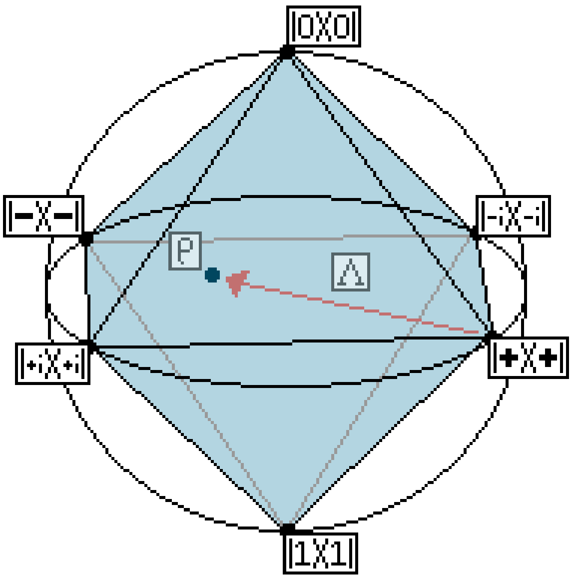

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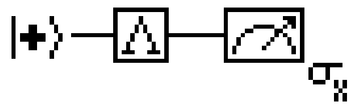

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$$= 0.3\,|+\rangle\langle+| + 0.7\,|-\rangle\langle-|$$

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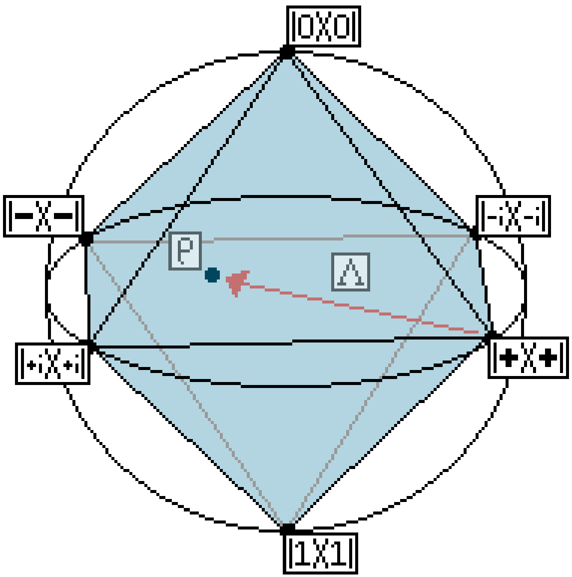

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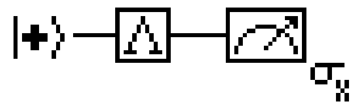

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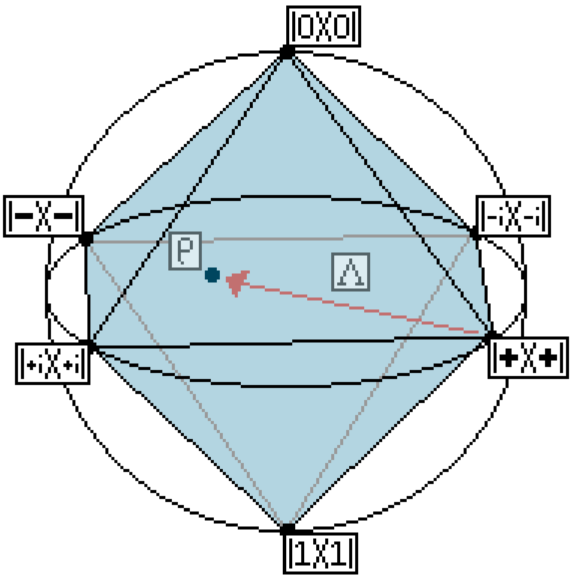

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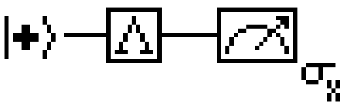

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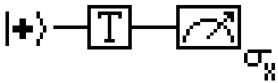
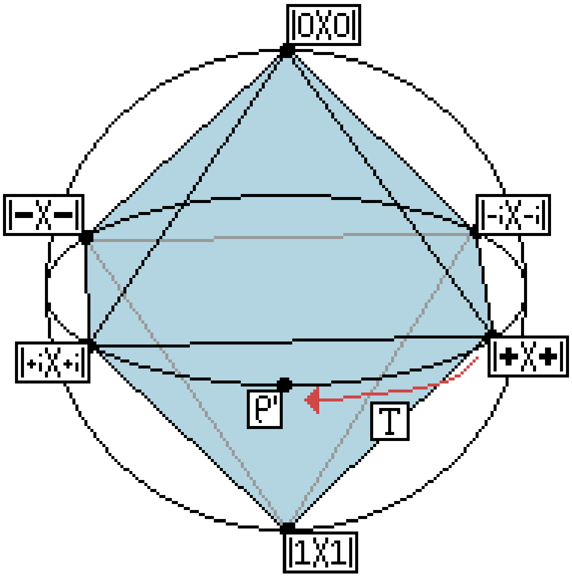
- $\mathrm{Tr}\left(\hat{\rho}\sigma_X\right)$ estimates $\mathrm{Tr}\left(\rho\sigma_X\right)$

Shuffling Stabilizer States (cont.)





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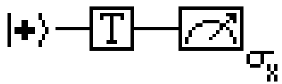
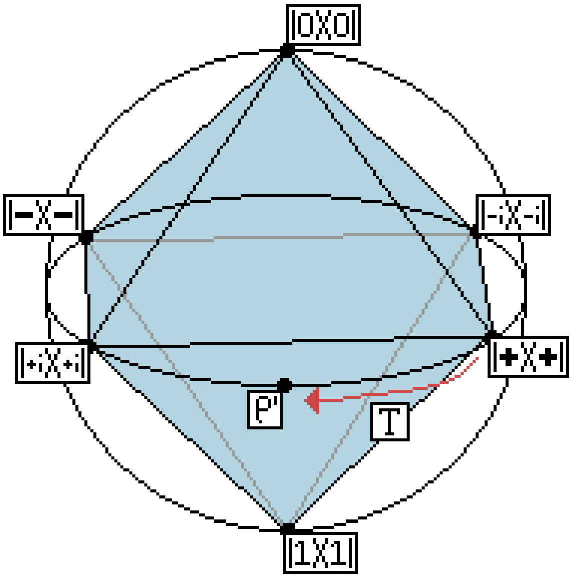
Shuffling Stabilizer States (cont.)





- What if output is not a stabilizer mixture, e.g. $T\,|+\rangle\langle+|\,T^\dagger = \rho'$?
- Can *still* write:

$$\rho' = \frac{\sqrt{2}+2}{4\sqrt{2}}\,|+\rangle\langle+| + \frac{\sqrt{2}+2}{4\sqrt{2}}\,|+i\rangle\langle+i|$$

$$+\frac{\sqrt{2}-2}{4\sqrt{2}}\,|-\rangle\langle-| + \frac{\sqrt{2}-2}{4\sqrt{2}}\,|-i\rangle\langle-i|$$

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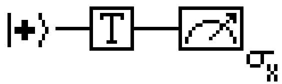

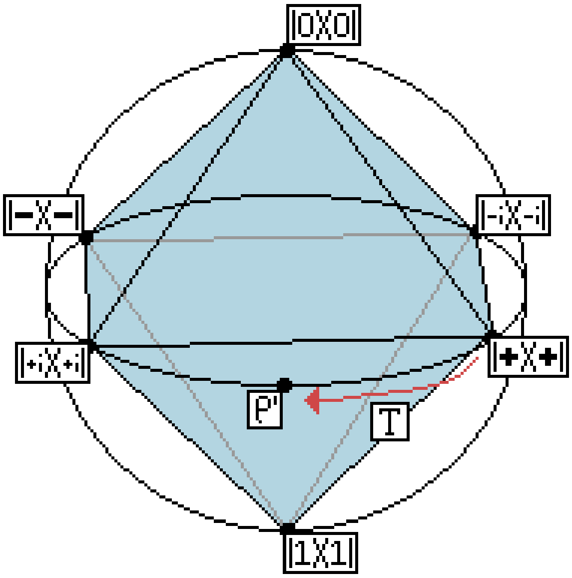

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$$\mathcal{R}(\rho') = \sum_{|\phi\rangle} |q_{|\phi\rangle}| = \sqrt{2}$$

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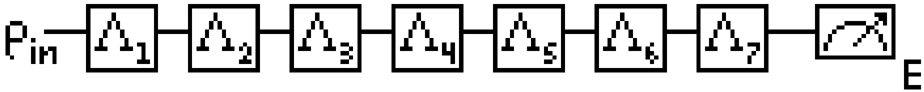
Runtime cost

- Repeated iteration:



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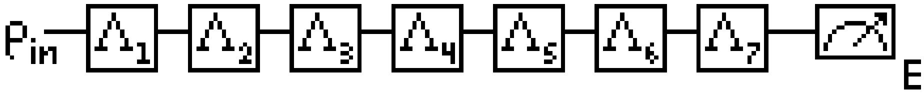

- Upper bound on sample magnitude:

$$|\text{sample}| \leq \mathcal{R}(\rho_{\text{in}}) \cdot \prod_i \max_{|\phi\rangle} \mathcal{R}\Big(\Lambda_i(|\phi\rangle\langle\phi|)\Big) \cdot \max_{|\phi\rangle} \text{Tr}\left(|\phi\rangle\langle\phi|\, E\right)$$

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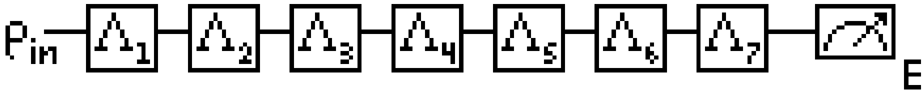

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- Hoeffding's inequality gives runtime: $\Pr(\text{error} \geq \varepsilon) \leq \delta$ if

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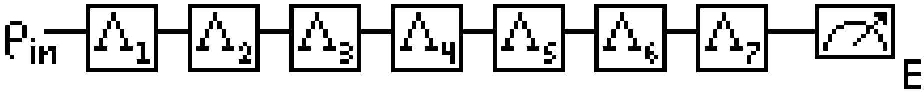

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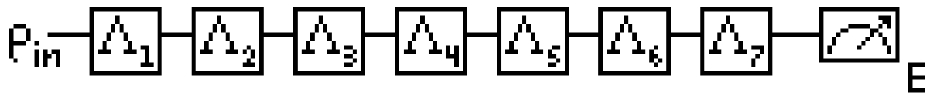

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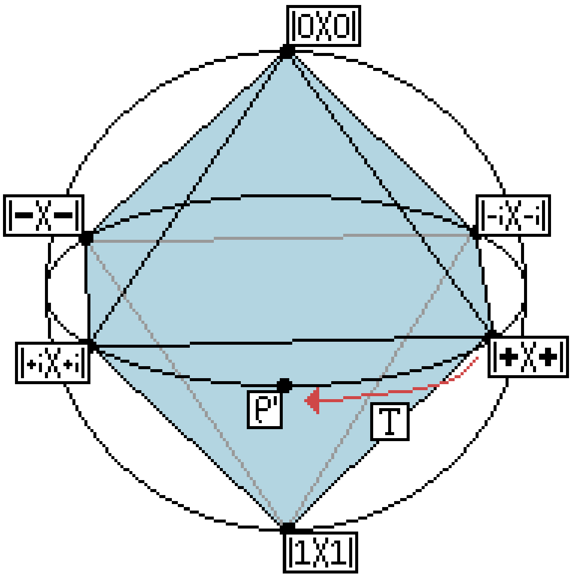
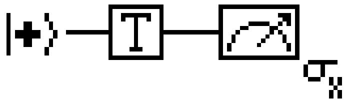
- Calculating $\mathcal{R}$ is hard. See Markus Heinrich's talk tomorrow!
- Lower bound $\mathcal{D}$ (aka st-norm): Phys. Rev. Lett. 118, 090501

$$\mathcal{R}(\rho) \geq \mathcal{D}(\rho) = 2^{-n} \sum_{\sigma_i} |\text{Tr}\left(\sigma_i \rho\right)|$$

Shuffling Paulis





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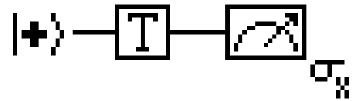

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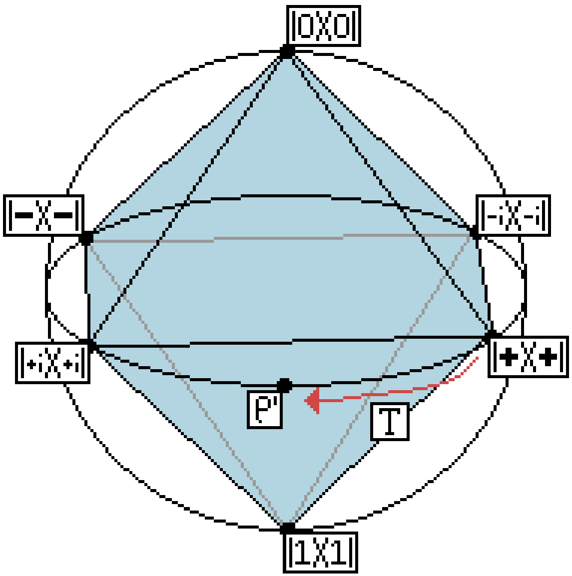

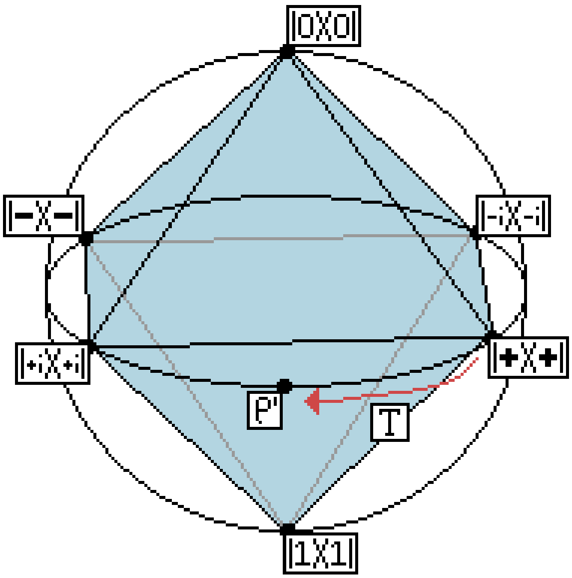

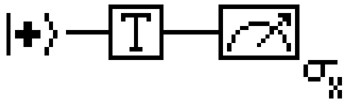

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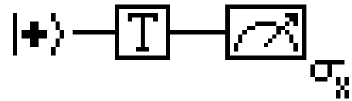

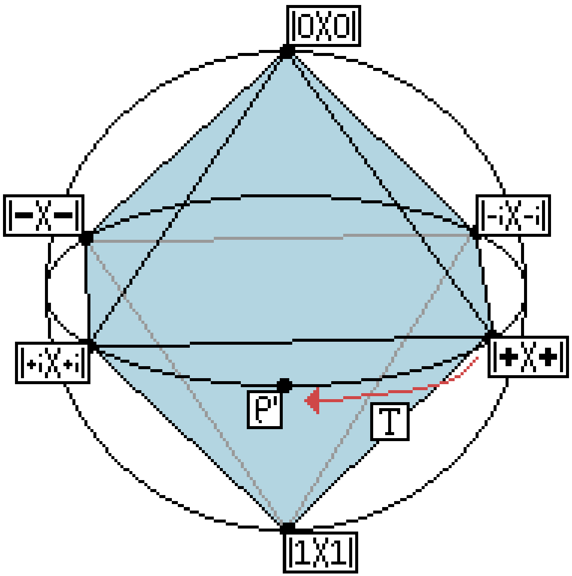

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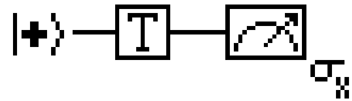

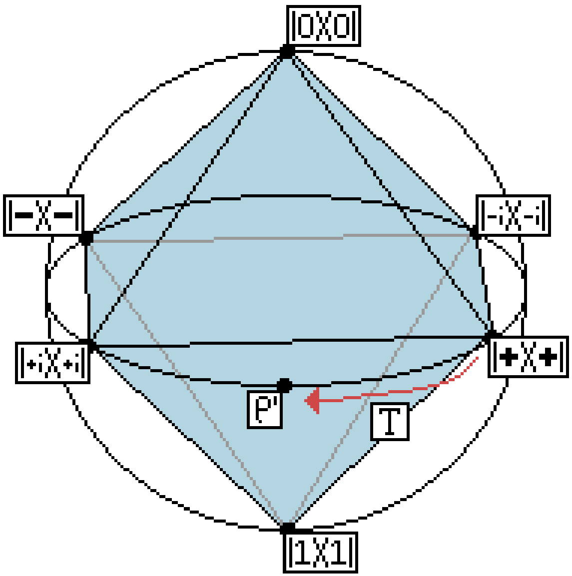

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- Sample:

$$\hat{\rho} = \begin{cases} \mathcal{D}(T\sigma_X T^\dagger)\sigma_X & \text{w.p. } (1/\sqrt{2})/\mathcal{D}(T\sigma_X T^\dagger) \\ \mathcal{D}(T\sigma_X T^\dagger)\sigma_Y & \text{w.p. } (1/\sqrt{2})/\mathcal{D}(T\sigma_X T^\dagger) \end{cases}$$

Outline for the rest of the talk

1. Properties of $\mathcal{D}$ and $\mathcal{R}$
2. Hyper-Octahedral States
3. Discarding Qubits
4. Performance in Practice
5. A Cartoon

Properties of $\mathcal{D}$ and $\mathcal{R}$

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- Shuffling Paulis can be faster than shuffling stabilizer states: Highly mixed states with $\mathcal{D}(\rho) \ll 1$ *improve* runtime!

Hyper-Octahedral States

- Families of states:
  - Stabilizer mixtures:

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  - Hyper-octahedral states:

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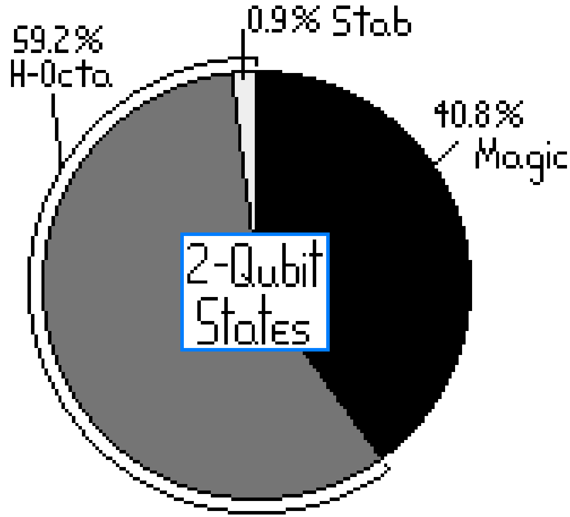

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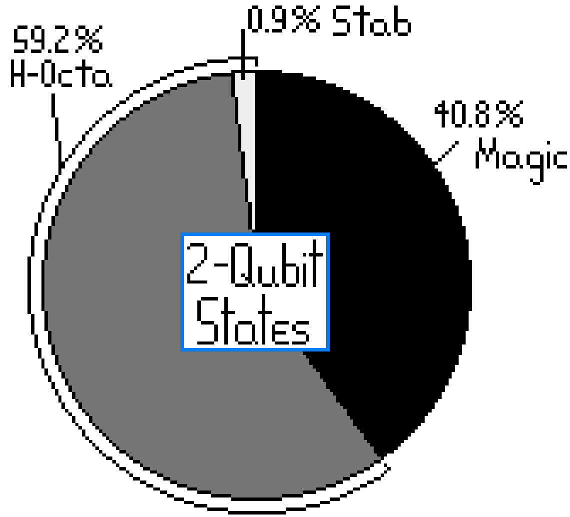
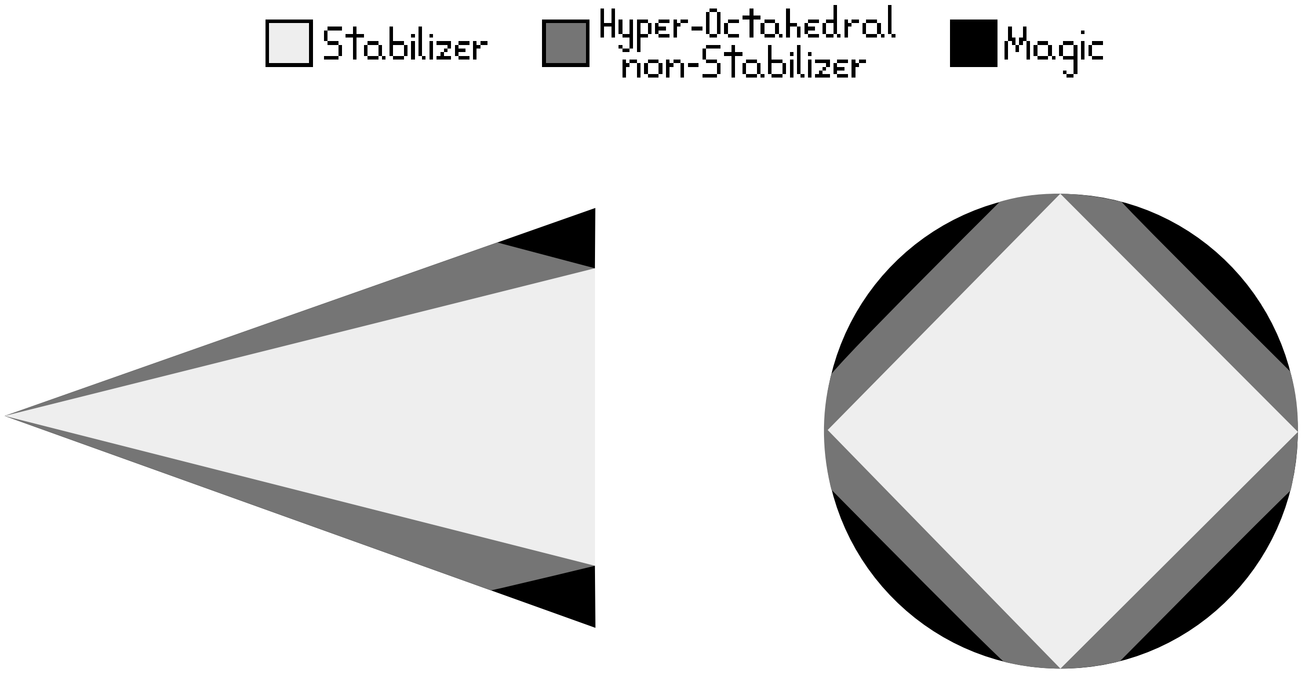


- Observation: In *multi-qubit systems* there exist hyper-octahedral states that are not stabilizer mixtures!
- There are a lot of these. (Based on 100 000 random 2-Qubit states)
- Previously known to exist for odd-dimensional systems
  e.g. New J. Phys. 15 039502, doi:10.1038/nature13460



$$\rho(x,y) = \frac{\sigma_{II}}{4} + x\sigma_{ZZ} + y(\sigma_{XX} + \sigma_{XY} + \sigma_{YX} - \sigma_{YY})$$

$$\rho(x,y) = \frac{\sigma_{II} + 0.8\sigma_{ZZ}}{4} + x(\sigma_{XX} - \sigma_{YY}) + y(\sigma_{XY} + \sigma_{YX})$$

Discarding Qubits

- Runtime $\sim \mathcal{O}\left(\max |\text{sample}|^2\right)$

$$|\text{sample}| \leq \mathcal{D}(\rho_{\text{in}}) \cdot \prod_j \max_{\sigma_i} \mathcal{D}\Big(\Lambda_j(\sigma_i)\Big) \cdot \max_{\sigma_i} \text{Tr}\,(\sigma_i E)$$

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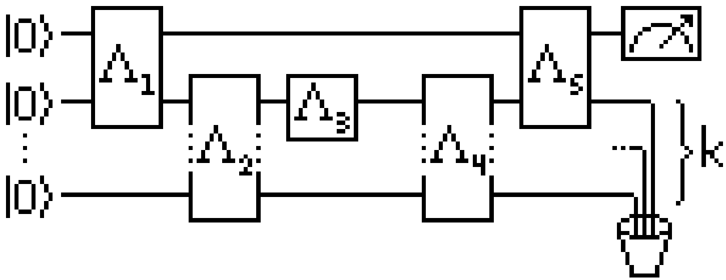

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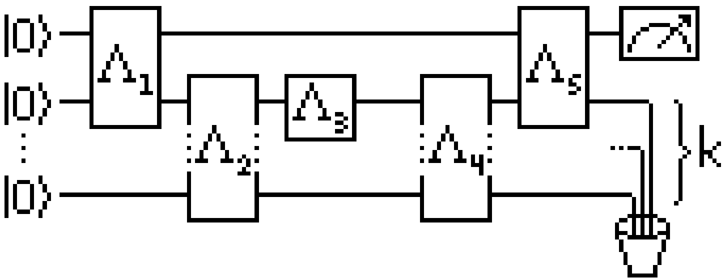
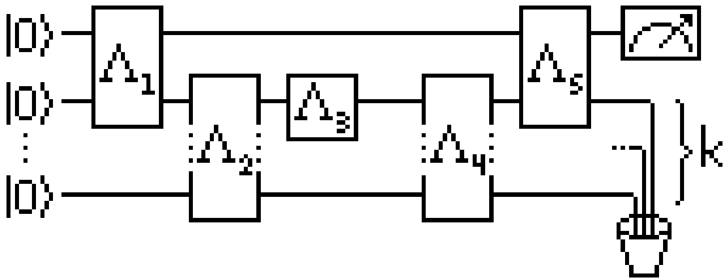


- $E = |0\rangle\langle 0| \otimes \sigma_I^{\otimes k}$ so $\max_{\sigma_i} \text{Tr}\left(\sigma_i E\right) = 2^k$, which can be large!

Discarding Qubits (contd.)

- We can usually recover polynomial runtime in # of qubits.
- Use back-propagation with $\Lambda^{-1}(E)$:

$$\mathrm{Tr}\left(\Lambda(\rho)E\right) = \mathrm{Tr}\left(\rho\Lambda^{-1}(E)\right)$$



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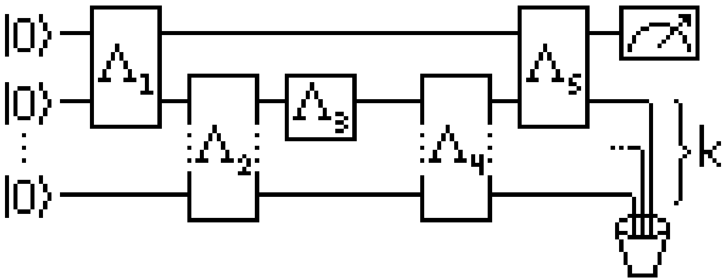

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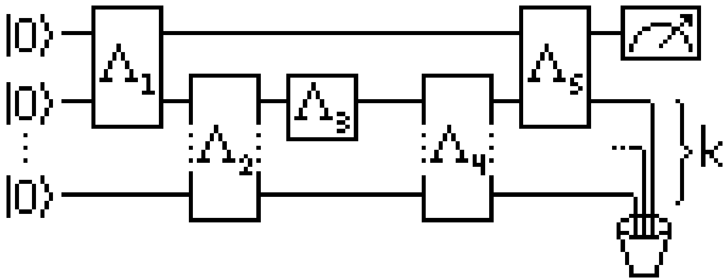
$$|\text{sample}| \leq \mathcal{D}(E)\cdot\prod_j \max_{\sigma_i}\mathcal{D}\left(\Lambda_j^{-1}(\sigma_i)\right)\cdot \max_{\sigma_i}\mathrm{Tr}\left(\sigma_i\rho_{\text{in}}\right)$$



Discarding Qubits (contd.)

- We can usually recover polynomial runtime in # of qubits.
- Use back-propagation with $\Lambda^{-1}(E)$:

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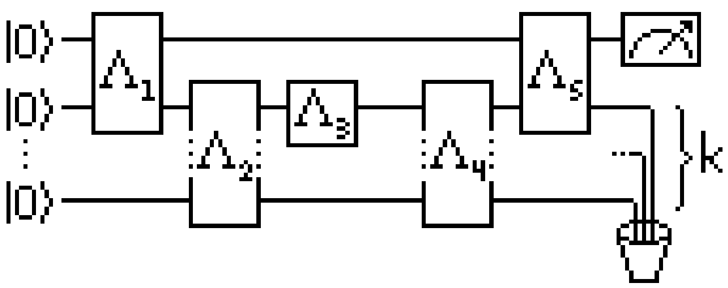

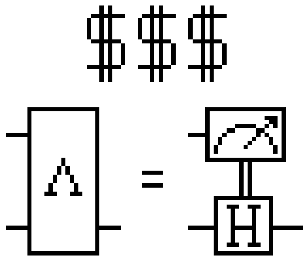

Performance in Practice

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  10 NVIDIA Tesla K40 GPUs take ≈ 40 billion samples in ≈ 30 min.

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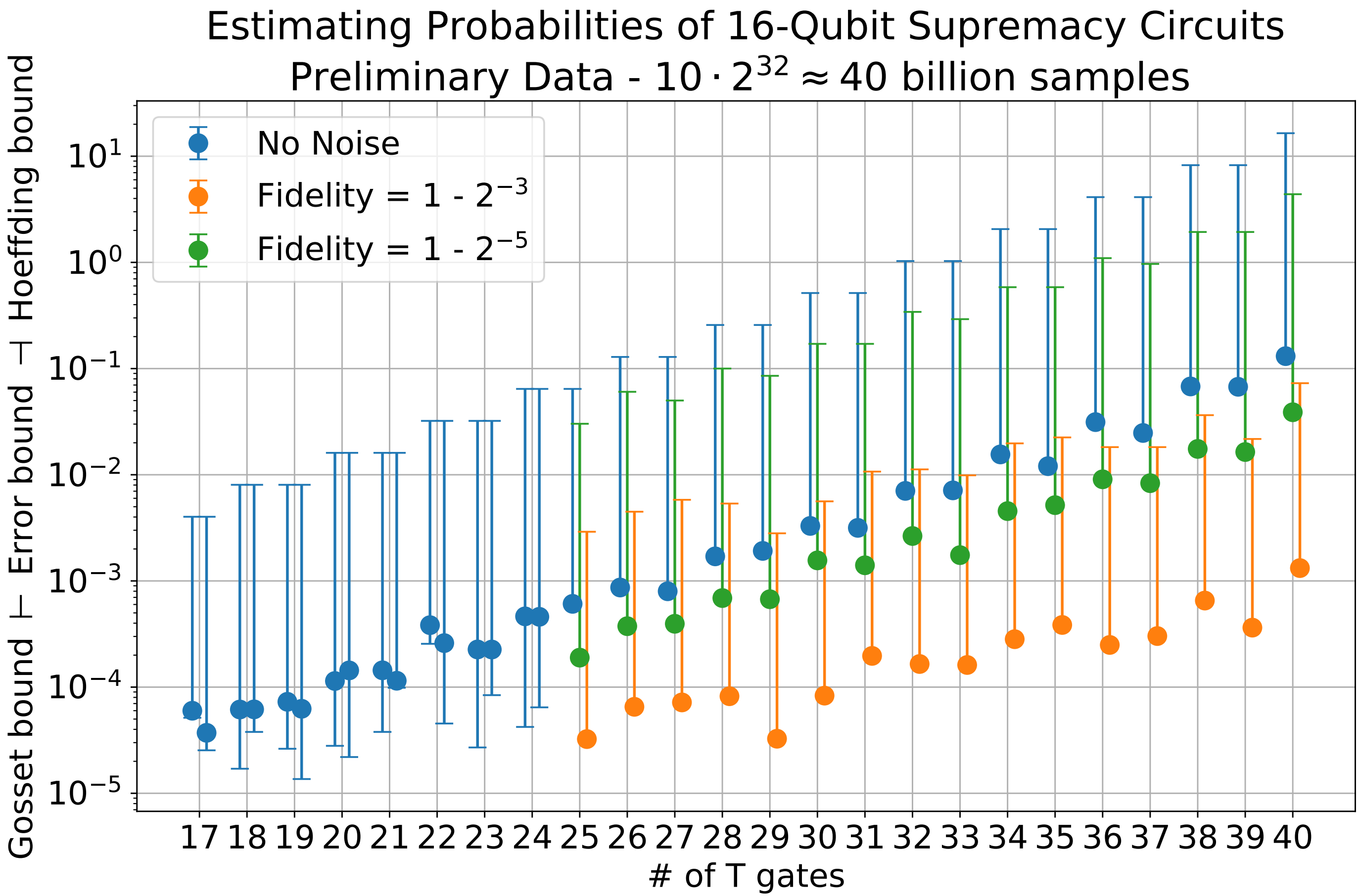
- Question: Is there an algorithm that:
  - is fast for Clifford circuits,
  - supports noisy channels,
  - *and* gives multiplicative error?

Performance in Practice (contd.)



A Cartoon

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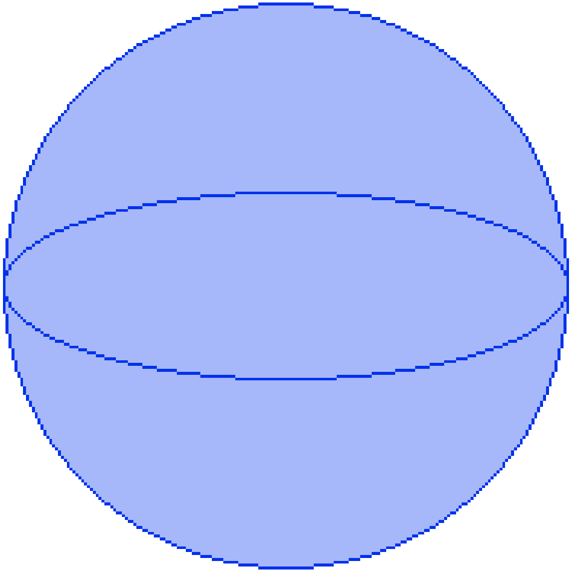

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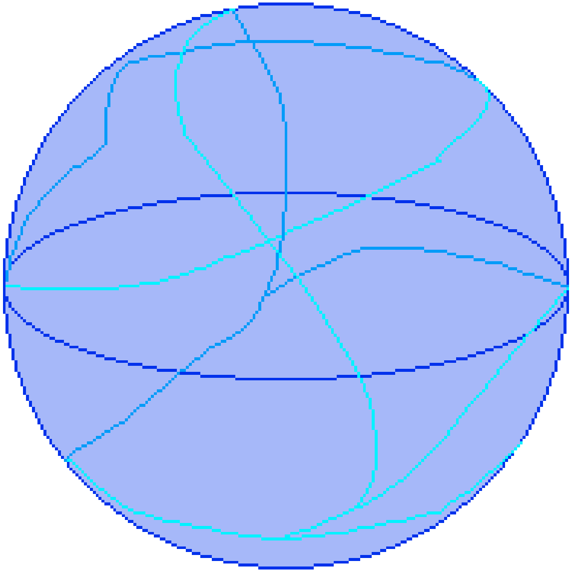

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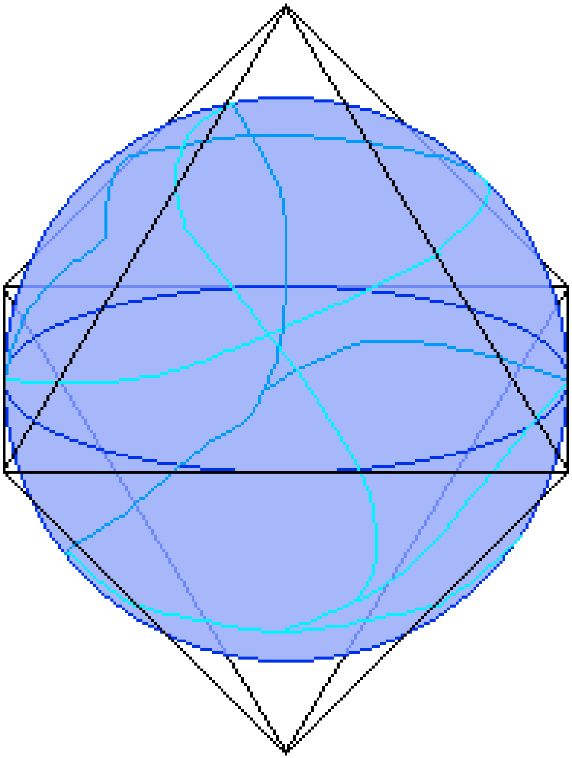

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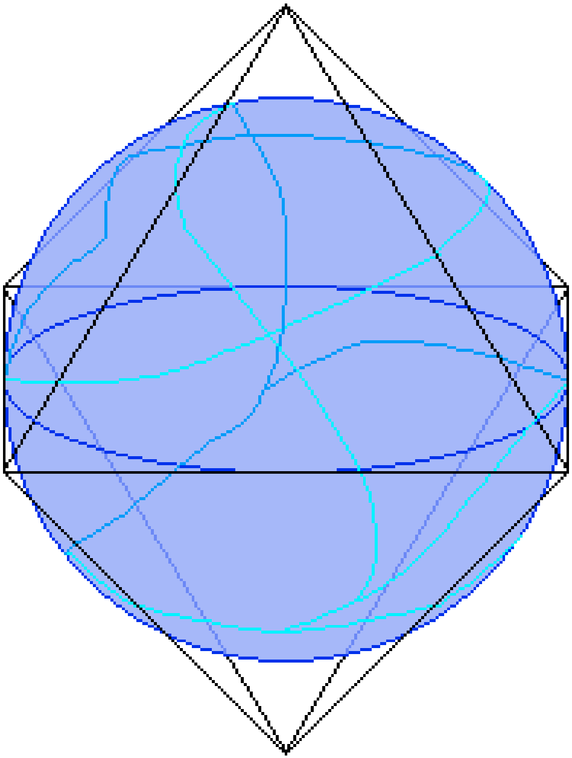

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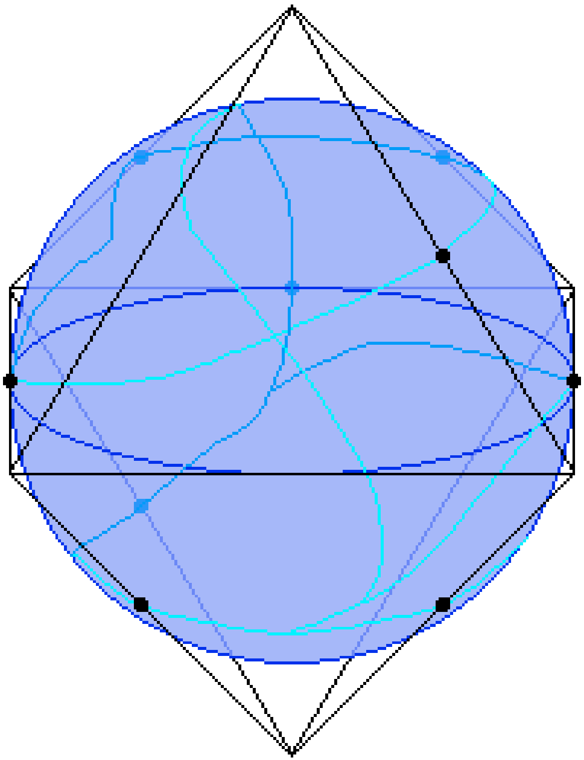

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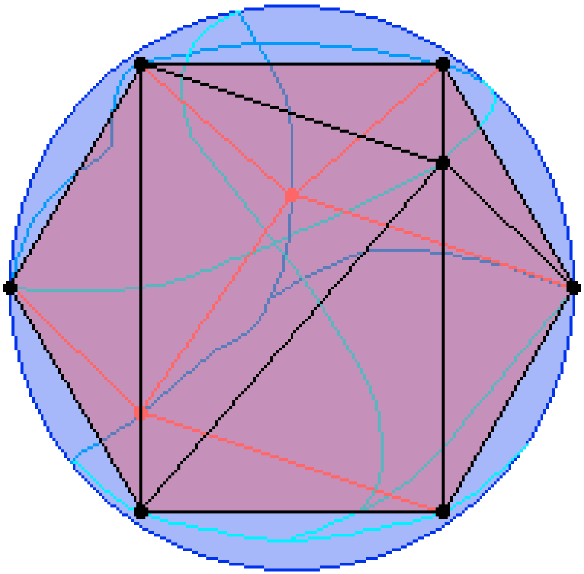

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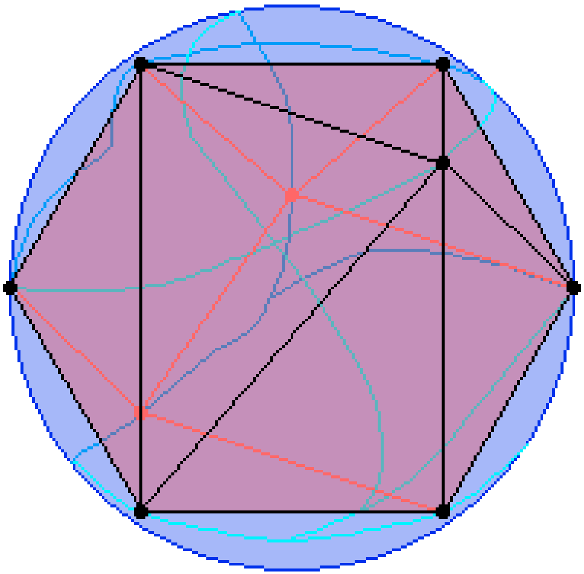

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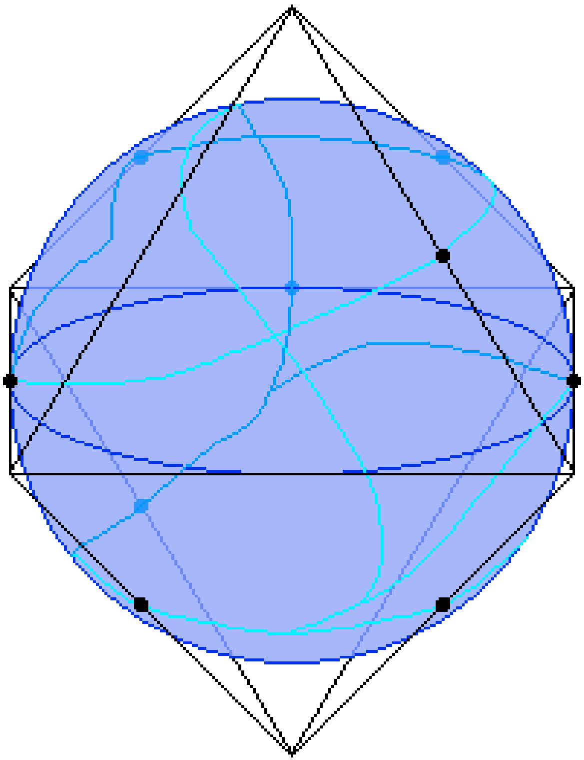

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